Deterministic and Random Selection of Variables in Local Search for SAT

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1 Introduction

The adaptive noise mechanism [2] was introduced in $Novelty$+ [1] to automatically adapt noise settings during the search, yielding the algorithm $adaptNovelty$+. The local search algorithm G^2WSAT deterministically selects the best promising decreasing variable to flip, if such variables exist [3]. However, the performance of G^2WSAT still depends on static noise settings. To avoid manual noise tuning, the adaptive noise mechanism [2] was integrated in G^2WSAT, resulting in the local search algorithm $adaptG^2WSAT$ [4, 5].

In this paper, we improve $adaptG^2WSAT$ and obtain a new local search algorithm called $adaptG^2WSAT+$. This new algorithm is different from $adaptG^2WSAT$ in two aspects. First, when there is no promising decreasing variable, $adaptG^2WSAT+$ uses $Novelty+$ instead of $Novelty$++ [3–5], to select a variable to flip from a randomly chosen unsatisfied clause c. Second, when promising decreasing variables exist, $adaptG^2WSAT+$ no longer flips the promising deceasing variable with the highest score among all promising decreasing variables, but chooses the least recently flipped promising decreasing variable among all promising decreasing variables to flip.

2 G^2WSAT and $adaptG^2WSAT$

Heuristics $Novelty$ [6], $Novelty$+ [1], and $Novelty$++ [3–5] select a variable to flip from a randomly selected unsatisfied clause c as follows.

$Novelty$(*p*)**:** Sort the variables in c by their scores, breaking ties in favor of the least recently flipped variable. Consider the best and second best variables from the sorted variables. If the best variable is not the most recently flipped one in c, then pick it. Otherwise, with probability p, pick the second best variable, and with probability 1-p, pick the best variable.

$Novelty+$(*p*, *wp*)**:** With probability wp, randomly pick a variable from c (random walk), with probability 1-wp, do as Novelty.

$Novelty$++(*p*, *dp*)**:** With probability dp (diversification probability), pick the least recently flipped variable in c, and with probability 1-dp, do as $Novelty$.

G^2WSAT [3] deterministically picks the promising decreasing variable with the highest score to flip, if such variables exist. If there is no promising decreasing variable, G^2WSAT uses a heuristic, such as $Novelty$++ [3–5], to pick a variable to flip from a randomly selected unsatisfied clause.

The adaptive noise mechanism of $adaptNovelty$+ [2] was integrated in G^2WSAT, resulting in the algorithm $adaptG^2WSAT$ [4, 5]. As a result, like $adaptNovelty$+, $adaptG^2WSAT$ is an algorithm in which no parameter has to be manually tuned to solve a new problem.

Algorithm: $adaptG^2WSAT+$(SAT-formula $\mathcal{F}$)

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1:  for try=1 to Maxtries do
2:      A← randomly generated truth assignment; p=0; wp=0;
3:      Store all decreasing variables in stack DecVar;
4:      for flip=1 to Maxsteps do
5:          if A satisfies F then return A;
6:          if |DecVar| > 0
7:          then y← the least recently flipped promising decreasing variable among
8:                   all promising decreasing variables in |DecVar|;
9:          else c ← randomly selected unsatisfied clause under A; y ← Novelty+(p, wp, c);
10:         A ← A with y flipped; Adapt p and wp;
11:         Delete variables that are no longer decreasing from DecVar;
12:         Push new decreasing variables into DecVar which are different from
13:          y and were not decreasing before y is flipped;
14: return Solution not found;
```

Fig. 1. Algorithm $adaptG^2WSAT+$

3 $adaptG^2WSAT+$

We improve $adaptG^2WSAT$ in two ways and obtain a new local search algorithm called $adaptG^2WSAT+$, which is described in Fig. 3. Since $adaptG^2WSAT$ deterministically uses promising decreasing variables, adding a small amount of randomness[3] to the search may help find a solution. Considering that the scores of promising decreasing variables are usually close and so such variables can improve the objective function roughly the same, flipping the least recently flipped promising decreasing variable can increase the mobility and coverage [7] of a local search algorithm in the search space.

4 Contest Implementation

For the SAT 2007 competition, in $adaptG^2WSAT+$, θ and ϕ, which are used to implement the adaptive noise mechanism, are set to $1/5$ and 0.1, respectively.

References

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[3] In general, random walk probability wp ranges from 0% to 10%.