Combining Adaptive Noise and Promising Decreasing Variables in Local Search for SAT

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Given a CNF formula $\mathcal{F}$ and an assignment A, the objective function that local search for SAT attempts to minimize is usually the total number of unsatisfied clauses in $\mathcal{F}$ under A. The score of a variable x with respect to A, $score_A(x)$, is the decrease of the objective function when it is flipped.

A variable x is said to be *decreasing* with respect to A if $score_A(x) > 0$. Promising decreasing variables are defined in [3] as follows:

1. Before any flip, i.e., when A is an initial random assignment, all decreasing variables with respect to A are promising.
2. Let x and y be two different variables and x be not decreasing with respect to A. If, after y is flipped, x becomes decreasing with respect to the new assignment, then x is a promising decreasing variable with respect to the new assignment.
3. A promising decreasing variable remains promising with respect to subsequent assignments in local search until it is no longer decreasing.

Let B be obtained from A by flipping x, and let x' be the best promising decreasing variable with respect to B. We define the promising score of x with respect to A as

$$pscore_A(x) = score_A(x) + score_B(x')$$

where $score_A(x)$ is the score of x with respect to A and $score_B(x')$ is the score of x' with respect to B.

If there are promising decreasing variables with respect to B, the promising score of x with respect to A represents the improvement in the number of unsatisfied clauses under A by flipping x and then x'. In this case, $pscore_A(x) > score_A(x)$.

If there is no promising decreasing variable with respect to B,

$$pscore_A(x) = score_A(x)$$

since one cannot know in advance which variable will be flipped for B (the choice of the variable to flip is made randomly.)

G^2WSAT [3] deterministically picks a promising decreasing variable to flip, if such variables exist. If there is no promising decreasing variable, G^2WSAT uses a heuristic, such as $Novelty$ [5], $Novelty$+ [1], $Novelty$++ [3], $Novelty$++', or $Novelty$++'p to pick a variable to flip from a randomly selected unsatisfied clause c.

$Novelty$**(**p**,** c**):** Sort the variables in clause c by their scores, breaking ties in favor of the least recently flipped variable. Consider the best and second best variables from the sorted variables. If the best variable is not the most recently flipped one in c, then pick it. Otherwise, with probability p, pick the second best variable, and with probability 1-p, pick the best variable.

$Novelty$**++(**p**,** dp**,** c**):** With probability dp (diversification probability), pick the least recently flipped variable in c, and with probability 1-dp, do as $Novelty$.

$Novelty$**++'(**p**,** dp**,** c**):** With probability dp (diversification probability), randomly pick a variable in c, after excluding the best and the second best variables by score in c, and with probability 1-dp, do as $Novelty$.

$Novelty$**+p(**p**,** wp**,** c**):** With probability wp (random walk probability), randomly pick a variable in c to flip. Otherwise, do as $Novelty$(p, c), except that if the best variable is more recently flipped than the second best, re-sort the best and the second best variables using their promising scores.

We propose three solvers for the SAT 2007 competition, all combining the adaptive noise mechanism [2] with G^2WSAT. The details of these solvers can be found in [4].

$adaptg2wsat$**:** if there are promising decreasing variables, flip the least recently flipped one. Otherwise, use $Novelty$++'(p, dp) to select the variable to flip,

$adaptg2wsat0$**:** if there are promising decreasing variables, flip the oldest one. Otherwise, use $Novelty$++'(p, dp) to select the variable to flip,

$adaptg2wsatp$**:** if there are promising decreasing variables, flip the least recently flipped one. Otherwise, use $Novelty$+p(p, dp) to select the variable to flip.

References

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